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Lecture (8)

Transmission line Models
&
Calculation

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1. Long line model

The ABCD parameters of the long transmission line can then be written as

$$A = D = \cosh \gamma l$$

$$B = Z_C \sinh \gamma l \ \Omega$$

$$C = \frac{\sinh \gamma l}{Z_C} \text{ mho}$$

2. Medium transmission lines

- Medium transmission lines are modeled with lumped shunt admittance.

- There are two different representations depending on the nature of the network:

2.1 Nominal (π).

2.2 Nominal (T).

2.1 Nominal (π)

- The ABCD parameters of the nominal (π) representation

$$A = D = \left(\frac{YZ}{2} + 1\right)$$

$$B = Z\ \Omega$$

$$C = Y\left(\frac{YZ}{4} + 1\right)\ \text{mho}$$

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2.2 Nominal (T)

- The ABCD parameters of the nominal (T) representation

$$A = D = \left(\frac{YZ}{2} + 1 \right)$$

$$B = Z\left(\frac{YZ}{4} + 1 \right)\Omega$$

$$C = Y \text{ mho}$$

3. Short line approximation

- The ABCD parameters of the short line representation

$$A = D = 1, B = Z\,\Omega \text{ and } C = 0$$

Example (1):

Example ⇨ Consider a 500 km long line for which the per kilometer line impedance and admittance are given respectively by $z = 0.1 + j0.5145\ \Omega$ and $y = j3.1734 \times 10^{-6}$ mho. Therefore

$$Z_C = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.1 + j0.5145}{j3.1734 \times 10^{-6}}} = \sqrt{\frac{0.5241\angle 79^\circ}{3.1734 \times 10^{-6}\angle 90^\circ}} = \sqrt{\frac{0.5241}{3.1734 \times 10^{-6}}}\angle\left(\frac{79^\circ - 90^\circ}{2}\right)$$

$$= 406.4024\angle -5.5^\circ\ \Omega$$

and

$$\gamma l = \sqrt{yz} \times l = \sqrt{0.5241 \times 3.1734 \times 10^{-6}} \times 500\angle\left(\frac{79^\circ + 90^\circ}{2}\right)$$

$$= 0.6448\angle 84.5^\circ = 0.0618 + j0.6419$$

We shall now use the following two formulas for evaluating the hyperbolic forms

$$\cosh(\alpha + j\beta) = \cosh\alpha\cos\beta + j\sinh\alpha\sin\beta$$
$$\sinh(\alpha + j\beta) = \sinh\alpha\cos\beta + j\cosh\alpha\sin\beta$$

Application of the above two equations results in the following values

Cont.

$$\cosh \gamma l = 0.8025 + j0.037 \text{ and } \sinh \gamma l = 0.0495 + j0.5998$$

Therefore from (2.43) to (2.45) the ABCD parameters of the system can be written as

$$A = D = 0.8025 + j0.037$$

$$B = 43.4 + j240.72\ \Omega$$

$$C = -2.01 \times 10^{-5} + j0.0015$$

ΔΔΔ

Example (2):

A single phase overhead transmission line delivers 1100 kW at 33 kV at 0·8 p.f. lagging. The total resistance and inductive reactance of the line are 10 Ω and 15 Ω respectively. Determine : (i) sending end voltage (ii) sending end power factor and (iii) transmission efficiency.

Solution.

Load power factor, $\cos\phi_R = 0{\cdot}8$ lagging

Total line impedance, $\vec{Z} = R + jX_L = 10 + j\,15$

Receiving end voltage, $V_R = 33 \text{ kV} = 33{,}000 \text{ V}$

$$\text{Line current, } I = \frac{kW \times 10^3}{V_R \cos\phi_R} = \frac{1100 \times 10^3}{33{,}000 \times 0{\cdot}8} = 41{\cdot}67 \text{ A}$$

$$\text{As } \cos\phi_R = 0{\cdot}8 \quad \therefore \quad \sin\phi_R = 0{\cdot}6$$

Cont.

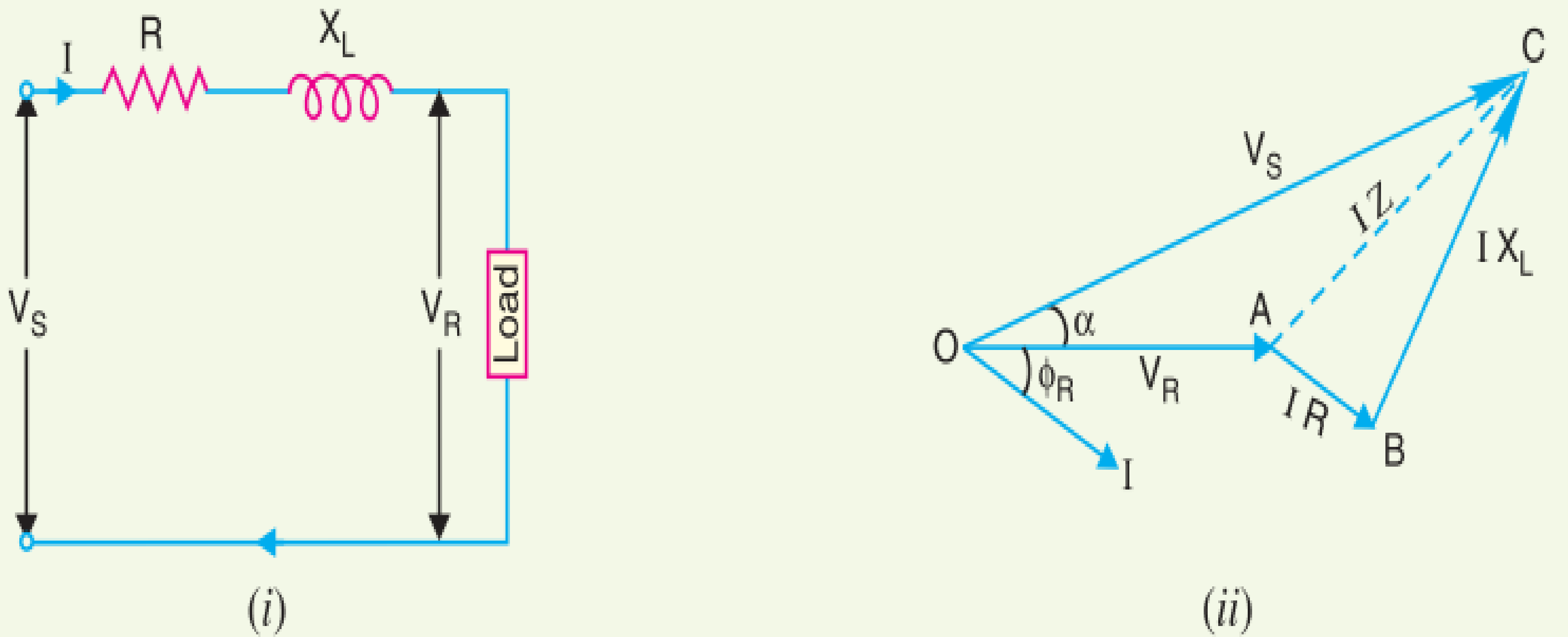

Fig. 10.5

The equivalent circuit and phasor diagram of the line are shown in Figs. 10.5 (*i*) and 10.5 (*ii*) respectively. Taking receiving end voltage $\vec{V}_R$ as the reference phasor,

Cont.

Performance of Single Phase Short Transmission Lines

$$\vec{V}_R = V_R + j\,0 = 33000 \text{ V}$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$$

$$= 41{\cdot}67\,(0{\cdot}8 - j\,0{\cdot}6) = 33{\cdot}33 - j\,25$$

(*i*) Sending end voltage, $\vec{V}_S = \vec{V}_R + \vec{I}\,\vec{Z}$

$$= 33{,}000 + (33{\cdot}33 - j\,25{\cdot}0)\,(10 + j\,15)$$

$$= 33{,}000 + 333{\cdot}3 - j250 + j500 + 375$$

$$= 33{,}708{\cdot}3 + j\,250$$

∴ Magnitude of $V_S = \sqrt{(33{,}708\cdot3)^2 + (250)^2} = \mathbf{33{,}709\ V}$

Cont.

(*ii*) Angle between $\vec{V_S}$ and $\vec{V_R}$ is

$$\alpha = \tan^{-1}\frac{250}{33{,}708\cdot 3} = \tan^{-1} 0{\cdot}0074 = 0{\cdot}42^{\circ}$$

$\therefore$ Sending end power factor angle is

$$\phi_S = \phi_R + \alpha = 36{\cdot}87^{\circ} + 0{\cdot}42^{\circ} = 37{\cdot}29^{\circ}$$

$\therefore$ Sending end p.f., $\cos\phi_S = \cos 37{\cdot}29^{\circ} =$ **0·7956 lagging**

(*iii*) Line losses $= I^2R = (41{\cdot}67)^2 \times 10 = 17{,}364 \text{ W} = 17{\cdot}364 \text{ kW}$

Output delivered $= 1100$ kW

Power sent $= 1100 + 17{\cdot}364 = 1117{\cdot}364$ kW

$\therefore$ Transmission efficiency $= \dfrac{\text{Power delivered}}{\text{Power sent}} \times 100 = \dfrac{1100}{1117\cdot 364} \times 100 =$ **98·44%**

Cont.

Note. V_S and ϕ_S can also be calculated as follows :

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R \text{ (approximately)}$$
$$= 33{,}000 + 41{\cdot}67 \times 10 \times 0{\cdot}8 + 41{\cdot}67 \times 15 \times 0{\cdot}6$$
$$= 33{,}000 + 333{\cdot}36 + 375{\cdot}03$$
$$= 33708{\cdot}39 \text{ V which is approximately the same as above}$$

$$\cos\phi_S = \frac{V_R\cos\phi_R + IR}{V_S} = \frac{33{,}000 \times 0{\cdot}8 + 41{\cdot}67 \times 10}{33{,}708{\cdot}39} = \frac{26{,}816{\cdot}7}{33{,}708{\cdot}39}$$
$$= 0{\cdot}7958$$

Example (3):

A (medium) single phase transmission line 100 km long has the following constants :

Resistance/km = 0·25 Ω ; *Reactance/km = 0·8 Ω*

Susceptance/km = 14 × 10^{-6} siemen ; *Receiving end line voltage = 66,000 V*

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Solution. Figs. 10.10 (*i*) and (*ii*) show the circuit diagram and phasor diagram of the line respectively.

Cont.

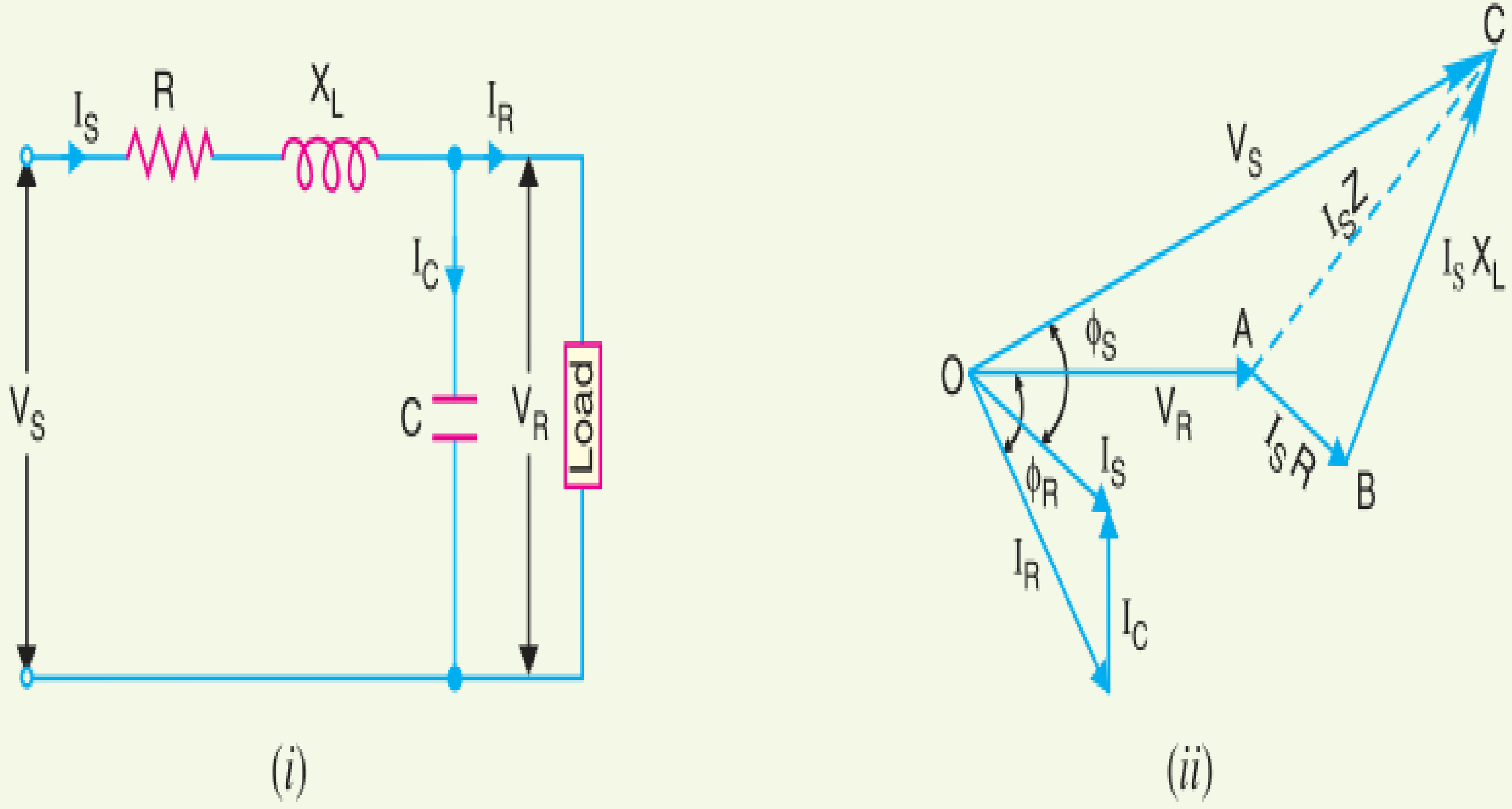

Fig. 10.10

Cont.

Total resistance, $R = 0{\cdot}25 \times 100 = 25\ \Omega$

Total reactance, $X_L = 0{\cdot}8 \times 100 = 80\ \Omega$

Total susceptance, $Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\ S$

Receiving end voltage, $V_R = 66{,}000\ \text{V}$

∴ Load current, $I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0 \cdot 8} = 284\ \text{A}$

$$\cos\phi_R = 0{\cdot}8\ ; \qquad \sin\phi_R = 0{\cdot}6$$

Taking receiving end voltage as the reference phasor [see Fig.10.10 (*ii*)], we have,

$$\vec{V_R} = V_R + j\,0 = 66{,}000\,V$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 284\,(0{\cdot}8 - j\,0{\cdot}6) = 227 - j\,170$

Cont.

Capacitive current, $\vec{I_C} = jY \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(*i*) Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (227 - j\,170) + j\,92$

$= 227 - j\,78$ ... (*i*)

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(*ii*) Voltage drop $= \vec{I_S}\,\vec{Z} = \vec{I_S}(R + j\,X_L) = (227 - j\,78)\,(\,25 + j\,80)$

$= 5{,}675 + j\,18,160 - j\,1950 + 6240$

$= 11{,}915 + j\,16{,}210$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$= 77{,}915 + j\,16{,}210$ ...(*ii*)

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

Cont.

(*iii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(*iv*) Referring to exp. (*i*), phase angle between $\vec{V}_R$ and $\vec{I}_S$ is :

$$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-0{\cdot}3436) = -18{\cdot}96^\circ$$

Referring to exp. (*ii*), phase angle between $\vec{V}_R$ and $\vec{V}_S$ is :

$$\theta_2 = \tan^{-1}\frac{16210}{77915} = \tan^{-1}(0 \cdot 2036) = 11 \cdot 50^\circ$$

∴ Supply power factor angle, $\phi_S = 18{\cdot}96^\circ + 11{\cdot}50^\circ = 30{\cdot}46^\circ$

∴ Supply p.f. $= \cos\phi_S = \cos 30{\cdot}46^\circ =$ **0·86 lag**

Example (4):

A 3-φ transmission line 200 km long has the following constants :

$$\text{Resistance/phase/km} = 0{\cdot}16\ \Omega$$

$$\text{Reactance/phase/km} = 0{\cdot}25\ \Omega$$

$$\text{Shunt admittance/phase/km} = 1{\cdot}5 \times 10^{-6}\ S$$

Calculate by rigorous method the sending end voltage and current when the line is delivering a load of 20 MW at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV.

Solution :

Total resistance/phase, $R = 0{\cdot}16 \times 200 = 32\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}25 \times 200 = 50\ \Omega$

Total shunt admittance/phase, $Y = j\,1{\cdot}5 \times 10^{-6} \times 200 = 0{\cdot}0003 \angle\ 90^\circ$

Series Impedance/phase, $Z = R + j\,X_L = 32 + j\,50 = 59{\cdot}4 \angle\ 58^\circ$

The sending end voltage V_S per phase is given by :

$$V_S = V_R \cosh \sqrt{Y\,Z} + I_R \sqrt{\frac{Z}{Y}} \sinh \sqrt{Z\,Y} \qquad \text{...}(i)$$

Now

$$\sqrt{Z\,Y} = \sqrt{59\cdot4 \angle 58° \times 0\cdot0003 \angle 90°} = 0\cdot133 \angle 74°$$

$$Z\,Y = 0\cdot0178 \angle 148°$$

$$Z^2 Y^2 = 0\cdot00032 \angle 296°$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{59\cdot4 \angle 58°}{0\cdot0003 \angle 90°}} = 445 \angle -16°$$

$$\sqrt{\frac{Y}{Z}} = \sqrt{\frac{0\cdot0003 \angle 90°}{59\cdot4 \angle 58°}} = 0\cdot00224 \angle 16°$$

$$\cosh \sqrt{Y\,Z} = 1 + \frac{Z\,Y}{2} + \frac{Z^2\,Y^2}{24} \text{ approximately}$$

$$= 1 + \frac{0\cdot0178}{2} \angle 148° + \frac{0\cdot00032}{24} \angle 296°$$

$$= 1 + 0\cdot0089 \angle 148° + 0\cdot0000133 \angle 296°$$

$$= 1 + 0\cdot0089\,(-0\cdot848 + j\,0\cdot529) + 0\cdot0000133\,(0\cdot438 - j\,0\cdot9)$$

$$= 0\cdot992 + j\,0\cdot00469 = 0\cdot992 \angle 0\cdot26°$$

Cont.

$$\sinh\sqrt{YZ} = \sqrt{YZ} + \frac{(YZ)^{3/2}}{6} \text{ approximately}$$

$$= 0{\cdot}133 \angle\, 74° + \frac{0{\cdot}0024 \angle\, 222°}{6}$$

$$= 0{\cdot}133 \angle\, 74° + 0{\cdot}0004 \angle\, 222°$$

$$= 0{\cdot}133\,(0{\cdot}275 + j\,0{\cdot}961) + 0{\cdot}0004\,(-\,0{\cdot}743 - j\,0{\cdot}67)$$

$$= 0{\cdot}0362 + j\,0{\cdot}1275 = 0{\cdot}1325 \angle\, 74°6'$$

Receiving end voltage per phase is

$$V_R = 110 \times 10^3/\sqrt{3} = 63508 \text{ V}$$

Receiving end current,

$$I_R = \frac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 131\ A$$

Cont.

Putting the various values in exp (*i*), we get,

$$V_S = 63508 \times 0{\cdot}992 \angle 0{\cdot}26^\circ + 131 \times 445 \angle -16^\circ 0' \times 0{\cdot}1325 \angle 74^\circ 6'$$
$$= 63000 \angle 0{\cdot}26^\circ + 7724 \angle 58^\circ 6'$$
$$= 63000\,(0{\cdot}999 + j\,0{\cdot}0045) + 7724\,(0{\cdot}5284 + j\,0{\cdot}8489)$$
$$= 67018 + j\,6840 = 67366 \angle 5^\circ 50' \text{ V}$$

Sending end line-to-line voltage $= 67366 \times \sqrt{3} = 116{\cdot}67 \times 10^3 \text{ V} =$ **116·67 kV**

The sending end current I_S is given by :

$$I_S = V_R \sqrt{\frac{Y}{Z}} \sinh \sqrt{Y\,Z} + I_R \cosh \sqrt{Y\,Z}$$

Putting the various values, we get,

$$I_S = 63508 \times 0{\cdot}00224 \angle 16^\circ \times 0{\cdot}1325 \angle 74^\circ 6' + 131 \times 0{\cdot}992 \angle 0{\cdot}26^\circ$$
$$= 18{\cdot}85 \angle 90^\circ 6' + 130 \angle 0{\cdot}26^\circ$$
$$= 18{\cdot}85\,(-0{\cdot}0017 + j\,0{\cdot}999) + 130\,(0{\cdot}999 + j\,0{\cdot}0045)$$
$$= 129{\cdot}83 + j\,19{\cdot}42 = 131{\cdot}1 \angle 8^\circ \text{ A}$$

∴ Sending end current = **131·1 A**